Problem 1. Determine all pairs of integers (x, y) which satisfy the equation

$$x^2 - y^2 = 5x.$$

Problem 2. Determine all prime numbers p for which the number

$$2p^2 + 5^{p-1}$$

is also a prime number.

Problem 3. Let ABC be a triangle, D the midpoint of AB and E a point on BC such that $BE = 2EC$. If $\angle CDA = \angle BAE$, prove that $\angle BAC = 90°$.

Problem 4. Alice and Bob construct a 2023-digit number as follows: The digits are chosen consecutively from left to right with Bob choosing every third digit and Alice choosing all other digits.

E.g. for the number

$$\overline{a_1 a_2 a_3 \cdots a_{2023}}$$

Alice chose firstly the digits a_1, a_2, then Bob chose the digit a_3, then Alice chose the digits a_4, a_5 e.t.c.

Alice wins if the number is a multiple of k, otherwise Bob wins. Determine which of the two players has a winning strategy in the following cases:

  (a) $k = 11$

  (b) $k = 15$