1.2 Second Selection Test

1.2.1 Problems

Problem 1. Determine all positive integers k, n for which it holds that

$$3 \cdot 2^k + 1 = n^2 .$$

Problem 2. Let a, b and c be positive integers such that

$$\frac{a^2 - a - c}{b} + \frac{b^2 - b - c}{a} = a + b + 2 .$$

Prove that $a + b + c$ is a perfect square.

Problem 3. Let (K_1, R_1) and (K_2, R_2), with $R_1 \neq R_2$, be circles which are externally tangent at the point A. Let (ε) be a common tangent of the two circles not passing through A. The perpendicular to the line (ε) through the point A meets the perpendicular bisector of K_1K_2 at a point B. Prove that $K_1K_2 = 2AB$.

Problem 4. On 2023 pieces of paper we have written the numbers $1, 2, 3, \ldots, 2023$ and placed them inside a box A. On another 2023 pieces of paper we have written the numbers $-1, -2, -3, \ldots, -2023$ and placed them inside a box B. Afterwards, we pick one piece of paper from box A and one piece of paper from box B, and we write the sum of those two numbers on the blackboard, until all pieces of paper have been picked.

If the product of the 2023 numbers which are written on the board is equal to 2023–2^n, for some integer n, determine all possible values of n.