Turkish JTST 2023
Day 1

1. Find all triples (n, k, p) such that n and k are integers, p is a prime number and

$$|6n^2 - 17n - 39| = p^k.$$

AoPS

2. Let ABC be an acute angled triangle and K, L points on AC, BC respectively such that $\angle AKB = \angle ALB$. P is intersection of AL and BK and Q is the midpoint of segment KL. Let T, S be the intersections of $(AL$ and $(BK$ with the circumcircle of ABC, respectively. Prove that TK, SL, PQ are concurrent.

AoPS

3. Find all functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ that satisfy $f(x + f(x)) = f(-x)$, $\forall, x \in \mathbb{R}$ and $f(x) \leq f(y)$ whenever $x \leq y$.

AoPS

4. Initially, Asli distributes 1000 balls to 30 boxes as she wishes. After that, Asli and Zehra make alternated moves which consists of choosing a box and taking a ball from the chosen box. Asli is the first on move. The one who takes the last ball from any box takes that box to herself. What is the maximum number of boxes can Asli guarantee to take herself regardless of Zehra's moves?

AoPS

Day 2

5. Prove that for all positive real numbers a, b, c the following inequality holds

$$\frac{a^4+1}{b^3+b^2+b} + \frac{b^4+1}{c^3+c^2+c} + \frac{c^4+1}{a^3+a^2+a} \geq 2$$

AoPS

6. A marble is placed on 33 unit squares of a 10×10 chessboard. After that, the number of marbles in the same row or column with that square is written on each of the remaining empty unit squares. (In the unit square situated on row r, column c, if it contains no marble, we write the sum between the number of marbles on row r and the number of marbles on column c.) What is the maximum value of the sum of the numbers written on the board?
AoPS

7. Let ABC be a triangle and let $D \in AB$, $E \in AC$ be such that $DE \parallel BC$. The circumcircle of triangle ABC meets again the circumcircles of triangles BDE and CDE at K and L, respectively. BK and CL intersect at T. Prove that TA is tangent to the circumcircle of ABC.

AoPS

8. For a prime number p, can the number of positive integers n that make the expression

$$\frac{n^3 + np + 1}{n + p + 1}$$

an integer be 777?

AoPS