Cyprus Mathematical Society

A′ Selection Test for under 15 ½ years old

«Ευκλείδης»

Date: 29/01/2022 Time: 10:00-14:30

Instructions

1. Solve **all** problems, **justifying** fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.

Problem 1. Find all integer values of x for which the value of the expression

$$x^2 + 6x + 33$$

is a perfect square.

(A natural number ν is called a **perfect square** if there is a natural numbber α such that $\nu = \alpha^2$).

Problem 2. Let $AB\Gamma\Delta$ be a square. Let E, Z be points on the sides $AB, \Gamma\Delta$ of the square respectively, such that $\Delta E \parallel BZ$. Assume that the triangles $\triangle EA\Delta$, $\triangle Z\Gamma B$ and the parallelogram $BE\Delta Z$ have the same area.

If the distance between the parallel lines ΔE and BZ is equal to 1, determine the area of the square.

Problem 3. If x,y are real numbers with $x + y \geqslant 0$, determine the minimum value of the expression

$$K = x^5 + y^5 - x^4y - xy^4 + x^2 + 4x + 7$$

For which values of x,y does K take its minimum value?

Problem 4. Consider the digits $1, 2, 3, 4, 5, 6, 7$.

(α) Determine the number of seven-digit numbers with distinct digits that can be constructed using the digits above.

(β) If we place all of these seven-digit numbers in increasing order, find the seven-digit number which appears in the 2022^{th} position.