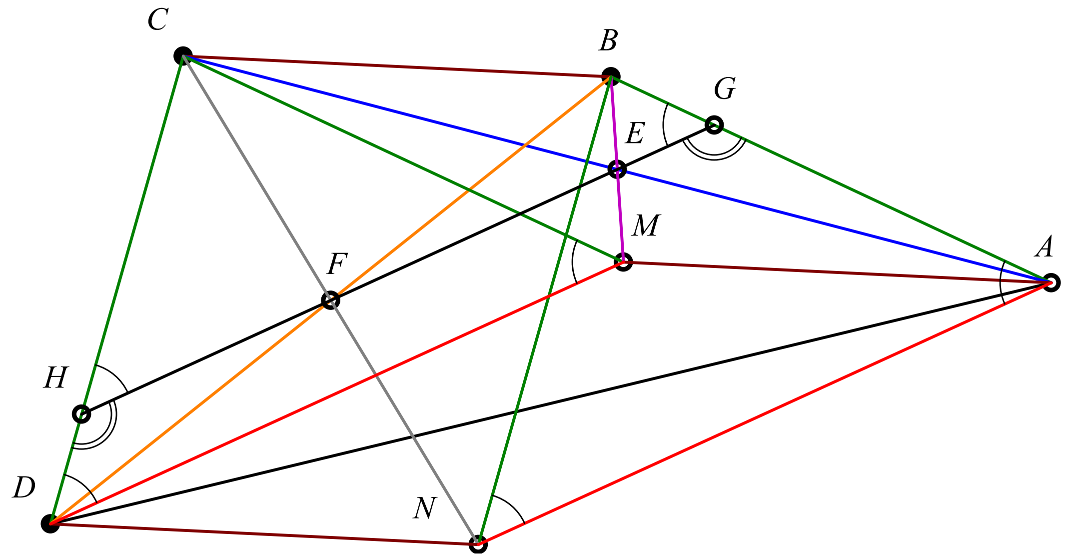
Problema săptămânii 273 (1-7 nov.2021):

Fie $ABCD$ – un patrulater convex, în care AB şi CD nu sunt paralele şi $[AB]\equiv[CD]$. Fie E şi F – mijloacele diagonalelor $[AC]$, respectiv $[BD]$. Dacă dreapta EF intersectează laturile $[AB]$ şi $[CD]$ în punctele G, respectiv H, demonstraţi că: $\widehat{AGH}\equiv\widehat{DHG}$.



SOLUŢIE (*Mihai Miculiţa*): Notând cu $M:=S_E(B)$ şi cu $N:=S_F(C)$, din:

$$\left.\begin{array}{r}\left.\begin{array}{r} M:=S_E(B)(ip)\Rightarrow[EM]\equiv[EB](1)\\ [EA]\equiv[EC](ip)\end{array}\right\}\Rightarrow MABC-paralelogram\Rightarrow[CM]\equiv\underline{[AB]}\\ \underline{[AB]}\equiv\underline{\underline{[CD]}}(ip)\\ \left.\begin{array}{r} N:=S_F(C)(ip)\Rightarrow[FN]\equiv[FC]\,(2)\\ [FB]\equiv[FD](ip)\end{array}\right\}\Rightarrow NBCD-paralelogram\Rightarrow[BN]\equiv\underline{\underline{[CD]}}\end{array}\right\}\Rightarrow$$

$$\Rightarrow[CM]\equiv[AB]\equiv[CD]\equiv[BN].\quad(3)$$

Pe de altă parte, din:

$$\left.\begin{array}{l}[EA]\equiv[EC](ip)\\ [FN]\equiv[FC](2)\end{array}\right\}\Rightarrow[EF]=linie\ mijlocie\ \hat{\imath}n\ \Delta CNA\Rightarrow\left\{\begin{array}{l}|AN|=2\cdot|EF|\quad(4)\\ \quad EF\parallel AN\quad(5)\end{array}\right.$$

$$\left.\begin{array}{l}[FD]\equiv[FB](ip)\\ [EM]\equiv[EB](1)\end{array}\right\}\Rightarrow[EF]=linie\ mijlocie\ \hat{\imath}n\ \Delta BDM\Rightarrow\left\{\begin{array}{l}|DM|=2\cdot|EF|\quad(6)\\ \quad EF\parallel DM;\quad(7)\end{array}\right.$$

iar din:

$$\left.\begin{array}{l}|AN|=2\cdot|EF|\,(4)\\ |DM|=2\cdot|EF|\,(6)\end{array}\right\}\Rightarrow[AN]\equiv[DM]\quad(8)$$

şi atunci, din:

$$\left.\begin{array}{r}[CM]\equiv[AB]\equiv[CD]\equiv[BN](3)\\ [AN]\equiv[DM](8)\end{array}\right\}\Rightarrow\Delta BNA\equiv\Delta CDM\,(LLL)\Rightarrow\widehat{BNA}\equiv\widehat{NAB}\equiv\widehat{CDM}\equiv\widehat{DMC}.\quad(9)$$

În fine, din:

$$\left.\begin{array}{r}HG(=EF)\parallel AN\ (5)\Rightarrow\widehat{CHG}\equiv\underline{\widehat{CDM}}\\ \underline{\widehat{CDM}}\equiv\underline{\underline{\widehat{NAB}}}(9)\\ HG(=EF)\parallel DM\ (7)\Rightarrow\widehat{HGB}\equiv\underline{\underline{\widehat{NAB}}}\end{array}\right\}\Rightarrow\widehat{CHG}\equiv\widehat{HGB}\Rightarrow m\left(\widehat{DHG}\right)=180^0-m\left(\widehat{CHG}\right)=$$

$$=180^0-m\left(\widehat{HGB}\right)=m\left(\widehat{AGH}\right)\Rightarrow\boxed{\widehat{DHG}\equiv\widehat{AGH}}.\ \blacksquare$$