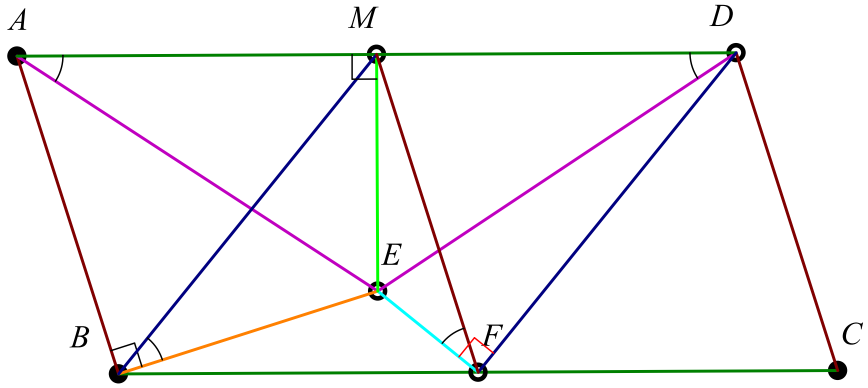
Problema BJ2 (Barajul I de selecţie a echipei Republicii Moldova, pt.OBMJ din 26 mai 2021):

În interiorul paralelogramului $ABCD$ se ia un punct E, astfel încât să avem:
$[AE]\equiv[DE]$ **şi** $BE\perp AB$. **Dacă** F **– este mijlocul laturii** $[BC]$, **aflaţi măsura unghiului** $\widehat{DFE}$.



SOLUŢIE (*Mihai Miculiţa*): Notând acum cu M – mijlocul laturii $[AD]$, din:

$$ABCD-paralelogram\,(ip)\Rightarrow\begin{cases}AD\parallel BC; & (1)\\ [AD]\equiv[BC]; & (2)\end{cases}$$ şi din:

$$\left.\begin{array}{r}\left.\begin{array}{r}|MA|=|MD|=\dfrac{|AD|}{2}(c.a.)\\ [AD]\equiv[BC]\,(2)\\ |FB|=|FC|=\dfrac{|BC|}{2}\,(ip)\end{array}\right\}\Rightarrow[MA]\equiv[MD]\equiv[FB]\\ AD\parallel BC\,(1)\end{array}\right\}\Rightarrow\begin{cases}ABFM-paralelogram\Rightarrow AB\parallel MF; & (3)\\ BFDM-paralelogram\Rightarrow BM\parallel DF. & (4)\end{cases}$$

În fine:

$$\left.\begin{array}{r}\left.\begin{array}{r}\left.\begin{array}{r}[AE]\equiv[DE](ip)\\ [MA]\equiv[MD](c.a)\end{array}\right\}\Rightarrow ME\perp AD\\ AD\parallel BC\,(1)\end{array}\right\}\Rightarrow ME\perp BF\,(=BC)\\ \left.\begin{array}{r}BE\perp AB\,(ip)\\ AB\parallel MF\,(3)\end{array}\right\}\Rightarrow BE\perp MF\end{array}\right\}\Rightarrow E-este\ ortocentrul\ \Delta MBF\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow EF\perp BM\\ BM\parallel DF\,(4)\end{array}\right\}\Rightarrow EF\perp DF\Rightarrow\boxed{m\left(\widehat{DFE}\right)=90^0}.\blacksquare$$