24th Junior Balkan Mathematical Olympiad (JBMO) 9 - 13 September 2020
Mathematical Society of South Eastern Europe (MASSEE)

Language: **English**

Friday, September 11, 2020

Problem 1. Find all triples (a, b, c) of real numbers that satisfy the system of equations:

$$a+b+c=\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \text{ and } a^2+b^2+c^2=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}.$$

Problem 2. Let $\triangle ABC$ be a right-angled triangle with $\angle BAC = 90^\circ$ and let E be the foot of the perpendicular from A on BC. Let $Z \neq A$ be a point on the line AB with $AB = BZ$. Let (c) be the circumcircle of the triangle $\triangle AEZ$. Let D be the second point of intersection of (c) with ZC and let F be the antidiametric point of D with respect to (c). Let P be the point of intersection of the lines FE and CZ. If the tangent to (c) at Z meets PA at T, prove that the points T, E, B, Z are concyclic.

Problem 3. Alice and Bob play the following game: Alice picks a set $A = \{1, 2, \ldots, n\}$ for some natural number $n \geqslant 2$. Then starting with Bob, they alternatively choose one number from the set A, according to the following conditions: initially Bob chooses any number he wants, afterwards the number chosen at each step should be distinct from all the already chosen numbers and should differ by 1 from an already chosen number. The game ends when all numbers from the set A are chosen. Alice wins if the sum of all the numbers that she has chosen is composite. Otherwise Bob wins. Decide which player has a winning strategy.

Problem 4. Find all prime numbers p and q such that

$$1+\frac{p^q-q^p}{p+q}$$

is a prime number.