The first selection test for JBMO 2003, April 12, 2003

JB1. Let $n \geq 2003$ be a positive integer such that the number $1 + 2003n$ is a perfect square. Prove that the number $n + 1$ is equal to the sum of 2003 positive perfect squares.

JB2. The positive real numbers a, b, c satisfy the relation $a^2 + b^2 + c^2 = 3abc$. Prove the inequality

$$\frac{a}{b^2c^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2} \geq \frac{9}{a+b+c}.$$

JB3. The quadrilateral $ABCD$ with perpendicular diagonals is inscribed in the circle with center O, the points M and N are the middle points of the sides $[BC]$ and $[CD]$ respectively. Find the value of the ratio of areas of the figures $OMCN$ and $ABCD$.

JB4. Let m and n be the arbitrary digits of the decimal system and a, b, c be the positive distinct integers of the form $2^m \cdot 5^n$. Find the number of the equations $ax^2 - 2bx + c = 0$, if it is known that each equation has a single real solution.

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JB5. Prove that each positive integer is equal to a difference of two positive integers with the same number of the prime divisors.

JB6. The real numbers x and y satisfy the equalities

$$\sqrt{3x}\left(1 + \frac{1}{x+y}\right) = 2, \qquad \sqrt{7y}\left(1 - \frac{1}{x+y}\right) = 4\sqrt{2}.$$

Find the numerical value of the ratio y/x.

JB7. The triangle ABC is isosceles with $AB = BC$. The point F on the side $[BC]$ and the point D on the side $[AC]$ are the feets of the internal bissectrix drawn from A and altitude drawn from B respectively so that $AF = 2BD$. Find the measure of the angle ABC.

JB8. In the rectangular coordinate system every point with integer coordinates is called laticeal point. Let $P_n(n, n+5)$ be a laticeal point and denote by $f(n)$ the number of laticeal points on the open segment (OP_n), where the point $O(0,0)$ is the coordinates system origine. Calculate the number $f(1) + f(2) + f(3) + \ldots + f(2002) + f(2003)$.