Mathematical Danube Competition

Juniors
October 26, 2019

Problem 1. Find the integer solutions of the equation

$$x^2(x^2+1) = 21^y - 1.$$

Problem 2. Prove that for any real numbers $a_1, a_2, \dots, a_n$, $n \in \mathbb{N}$, there exists a real number x such that the numbers $x + a_1$, $x + a_2$, $\dots$, $x + a_n$ are all irrational.

Problem 3. One a line, there are 51 positive integers whose sum is 100. Prove that, for all positive integers k, $1 \leq k \leq 99$, one can find either a succession of numbers on the line whose sum is k, or a succession of numbers whose sum is $100-k$.

Problem 4. Consider a cyclic quadrilateral $ABCD$ and let M and N be the midpoints of the diagonals AC and BD, respectively. If $\angle AMB = \angle AMD$, prove that $\angle ANB = \angle BNC$.