Problema săptămânii 158

Fie $a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_{2n-1} \geq a_{2n} \geq 0$ numere reale cu proprietatea $\sum_{i=1}^{2n} a_i = 1$. Arătaţi că

$$a_1a_2 + 3a_3a_4 + 5a_5a_6 + \ldots + (2n-1)a_{2n-1}a_{2n} \leq \frac{1}{4}.$$

Când are loc egalitatea?

Problem of the week no. 158

Let $a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_{2n-1} \geq a_{2n} \geq 0$ such that $\sum_{i=1}^{2n} a_i = 1$. Prove that

$$a_1a_2 + 3a_3a_4 + 5a_5a_6 + \ldots + (2n-1)a_{2n-1}a_{2n} \leq \frac{1}{4}.$$

When does the equality hold?