«Ευκλείδης»

Date: 20/04/2019	**Time duration: 10:00-14:30**

Instructions:
1. Solve all the problems by showing your work .
2. Write with blue or black ink . (For figures you can use pencil)
3. Do not use corrector liquid   (Tipp-ex).
4. Do not use calculators   .

Problem 1 : (α) If x is a positive real number, prove that
$$\frac{x^{12}-1}{4} \geq \frac{x^3-1}{x}$$
(β) If α, β are positive real numbers, prove that
$$\frac{\alpha^{12}+\beta^{12}}{4} + \frac{1}{\beta} + \frac{1}{\alpha} \geq \alpha^2 + \beta^2 + \frac{1}{2}$$

Problem 2 : Let $AB\Gamma\Delta$ be a rectangle such that $AB > 2B\Gamma$. Let M be a point on the side AB such that $AM = B\Gamma$ and let N be a point on the ray ΓB such that $\Gamma N = MB$. The line through A parallel to ΓM meets $\Delta\Gamma$ at P. Let K be the point of intersection of the lines ΓM and AN. Prove that
(ι) $AP = PN$
(ιι) Points A, K, M and Δ belong on the same circle.

Problem 3 : In a basketball tournament the only participants are teams from Limassol and Nicosia. The number of teams from Nicosia is 9 more than the number of teams from Limassol. Every two teams played each other exactly once. The winner took 1 point, the loser 0 points and there were no draws. The total number of points gathered by the teams from Nicosia were 9 times as many as the total number of points gathered by the teams from Limassol. Determine the largest possible number of wins of the most successful team from Limassol.

Problem 4 : Find 10 distinct prime numbers all of which divide the number
$$A = 11111^{60} - 10009^{60}$$