MOCK TEST FOR JBMO 2017

Test 1

April 21st, 2017

Problem 1. Find all pairs (a, b) of positive integers such that $(a^2 + b)(b^2 + a)$ is a power of 2.

Problem 2. Let a, b, c be positive real numbers such that $ab + bc + ca = 3abc$. Prove that

$$\frac{1}{\sqrt{2a^2 + 5ab + 2b^2}} + \frac{1}{\sqrt{2b^2 + 5bc + 2c^2}} + \frac{1}{\sqrt{2c^2 + 5ca + 2a^2}} \leq 1.$$

Problem 3. Let $ABCD$ be a cyclic quadrilateral and P is the intersection of AC, BD. Point Q is taken on the side BC such that $PQ \perp AC$ and point T is taken on PQ such that $DT \perp DA$.

(a) Denote by I the circumcenter of triangle APD. Prove that $ID = IT$.

(b) Denote by J the circumcenter of triangle BQD. Prove that $IJ \parallel AD$.

Problem 4. Let be given a table of size 2016×2017 with each cell contains $+$ sign. Someone changes the sign of the cell from $+$ to $-$, from $-$ to $+$ with the following rules:

(i) Each cell of the i-th row was changed i times, $1 \leq i \leq 2016$.

(ii) Each cell of the j-th column was changed $3j + 1$ times, $1 \leq j \leq 2017$.

After all operations, how many $+$ signs will remain on the table?

Time allowed: 4 hours and 30 minutes.

Each problem is worth 10 points.