CYPRUS MATHEMATICAL SOCIETY
B΄ SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 10/02/2019 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work.
2. Write with blue or black ink. (You may use pencil for figures)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.

Problem 1. Let p be a prime number and let β be an integer such that:

- The number $2019+\beta$ is a multiple of p
- The number $2019^3+\beta^3$ is a multiple of p^2
- The number p^2 does not divide $2019+\beta$.

Prove that the number $2019^3+\beta^3$ is a multiple of p^3

Problem 2: Let μ, ν be positive integers such that the number $A=\mu^3+\nu^3-(\mu+\nu)^2$ is also a positive integer.
(α) Prove that $A=(\mu+\nu)(\mu^2+\nu^2-\mu\nu-\mu-\nu)$
(β) Determine the minimum possible value of A.

Problem 3: Let $\varDelta AB\Gamma$ be an isosceles triangle with $\Gamma A=\Gamma B$ and $\angle A\Gamma B>90°$. Let E be the point of intersection of the perpendicular bisector of $A\Gamma$ with the internal bisector of angle $\angle B$ of the triangle $\varDelta AB\Gamma$. The circle with diameter AE meets the line $A\Gamma$ at Z. If the tangent to the circle at point Z meets line AB at Θ, prove that $AZ=A\Theta$.

Problem 4: In a meeting of 100 people, every person hates exactly one other person. (Hating is not necessarily mutual.)
(α) Prove that we can choose 34 people such that none of them hates another one of them.
(β) Find an example for which however we choose 35 people, one of those will hate another one of those.