ΚΥΠΡΙΑΚΗ ΜΑΘΗΜΑΤΙΚΗ ΕΤΑΙΡΕΙΑ

Α΄ ΔΙΑΓΩΝΙΣΜΟΣ ΕΠΙΛΟΓΗΣ ΚΑΤΩ **ΤΩΝ 15 1/2 ΕΤΩΝ**

«Ευκλείδης»

Ημερομηνία: 12/01/2019 **Ώρα εξέτασης: 10:00-14:30**

INSTRUCTIONS:

1. Solve all the problems justifying fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid is not permitted.
4. Calculators are not permitted.

Problem 1: (α) Factor the polynomial $x^4 + 4$ into a product of two non-constant polynomials with integer coefficients.

(β) Determine all pairs of positive integers (α, β) for which the expression

$$\alpha^4 + 4\beta^4$$

is a prime number.

Problem 2: Determine all triples of real numbers (α, β, γ) for which all of the following conditions hold:

(i) $\alpha\beta\gamma = 1$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \alpha + \beta + \gamma$

(iii) $\beta - \alpha = 1$

Problem 3: Let $AB\Gamma$ be an acute-angled triangle. Let M be the midpoint of $B\Gamma$ and suppose that $AB < AM$. Let Δ be a point on AM such that $AB = A\Delta$. Let (δ) be the bisector of $\angle BA\Delta$ and let E be the point of intersection of (δ) with $B\Delta$. The perpendicular line from Γ to (δ) meets AM at O and (δ) at N. Let K, T be the midpoints of EN and $O\Gamma$ respectively. If the parallel through M to the line (δ) meets ΓN at Z, prove that $KZ = MT$.

Problem 4 The following numbers are written on the blackboard

2018 ones, 2019 twos, and 2020 threes

In every move the player erases two distinct numbers are writes in their place the third number. (The player erases two numbers and writes only one number.) The game ends when the player cannot make any move.

Each one of Andrew, Vasilis and George plays the game separately on his own until it ends. At the end of his own game, Andrew wants to leave on the blackboard the number one only once (with no other number) Vasilis wants to leave the number two only once (with no other number) and George wants to leave the number three only once (with no other number). Explain fully which of the three players can achieve their target.