Alternative solution to problem **A1**

From the inequality between the quadratic mean and the arithmetic mean (or CBS) we have

$$\sqrt{\frac{a^2+b^2+1^2+1^2}{4}} \geq \frac{a+b+1+1}{4}.$$

We obtain that $\sqrt{a^2+b^2+2} \geq \dfrac{a+b+2}{2}$ and two other similar inequalities.
Adding them together, we obtain that

$$\sqrt{a^2+b^2+2}+\sqrt{b^2+c^2+2}+\sqrt{c^2+a^2+2} \geq a+b+c+3.$$

Using the AM-GM inequality and the condition $a+b+c+ab+ac+bc+abc=7$ we obtain

$$(a+1)+(b+1)+(c+1) \geq 3\sqrt[3]{(a+1)(b+1)(c+1)}=6.$$

Alternative solution to problem **A4**
Write the given inequality as $(x+y+z)(xy+yz+zx)-9xyz \geq 2(x+y+z)$, or $x(y-z)^2+y(z-x)^2+z(x-y)^2 \geq 2(x+y+z)$.
Because of the simmetry of the inequality, we may assume $x<y<z$ and consider $a,b \in \mathbb{N}$ such that $y=x+a$, $z=y+b$. The last inequality becomes

$$xb^2+(x+a)(a+b)^2+(x+a+b)a^2 \geq 2(3x+2a+b). \qquad (*)$$

Except when $a=1$, when have $x\big(b^2+a^2+(a+b)^2\big) \geq 6x$, $a(a+b)^2 \geq 4a$ and $a^2(a+b) \geq 2b$. Adding these three inequalities proves $(*)$.
In case $a=1$, $(*)$ becomes $xb^2+(x+1)(1+b)^2 \geq 5x+3+b$. This follows from $x(2b^2+2b+1) \geq 5x$ and $b^2+2b+1 \geq b+3$.
Equality holds when $a=b=1$.