CYPRUS MATHEMATICAL SOCIETY
C΄ SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 24/03/2018 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work.
2. Write with blue or black ink. (You may use pencil for figures)
3. Do not use corrector liquid (Tipp-ex).
4. Do not use calculators.

Problem 1: If α, β, γ positive real numbers prove that

$$\frac{(\alpha^2\gamma + \beta)(\beta^2\alpha + \gamma)(\gamma^2\beta + \alpha)}{\alpha^2\beta^2\gamma^2} \geq 8$$

When holds the equality;

Problem 2: Triangle $\triangle AB\Gamma$ is inscribed in the circle (O, R). Let Δ be the antidiametric point of A on the circle (O, R). Let Δ_1 be the symmetric point of Δ on the line AB and Δ_2 be the symmetric point of Δ on the line $A\Gamma$. Let I be the point of intersection of $A\Gamma$ and $B\Delta$, let K be the midpoint of $\Delta_1\Delta_2$, and let N το be the midpoint of $B\Gamma$. Prove that:
(α) $\angle\Delta_1 AI = \angle\Delta_1\Delta_2 I$ and
(β) the line through N parallel to $A\Delta$ passes through the midpoint of AK.

Problem 3: The positive integers α, β are defined by

$$\alpha = 2^{2018} + 3^{2018} + 4^{2018} + 5^{2018}\,, \quad \beta = \frac{3^{2019} - 3}{2}$$

Determine the units digit of the number $= \alpha^2 + \beta^2 + \alpha\beta$.

Problem 4: We have two piles with 2000 and 2018 coins respectively. Anna and Bob take alternate turns making the following moves: The player whose turn is to move picks a pile with at least two coins, removes from that pile t coins for some $2 \leq t \leq 4$, and adds to the other pile 1 coin. The players can, if they wish, choose a different t and a different pile at each turn, and the player who cannot make a move loses. If Anna plays first, determine which player has a winning strategy.