ΚΥΠΡΙΑΚΗ ΜΑΘΗΜΑΤΙΚΗ ΕΤΑΙΡΕΙΑ

Α΄ ΔΙΑΓΩΝΙΣΜΟΣ ΕΠΙΛΟΓΗΣ ΚΑΤΩ **ΤΩΝ 15 1/2 ΕΤΩΝ**

«Ευκλείδης»

Ημερομηνία: 13/01/2018 **Ώρα εξέτασης: 10:00-14:30**

INSTRUCTIONS:

1. Solve all the problems justifying fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid is not permitted.
4. Calculators are not permitted.

Problem 1: A school has 218 students. A survey at the school found out that:

i. 140 students have a bicycle,
ii. 159 students have a mobile phone,
iii. 181 students have a soccer ball, and
iv. 176 students have a PC.

How many of the students definitely have a bicycle, and a mobile phone, and a soccer ball, and a PC?

Problem 2: Let

$$A = \frac{\mu}{\nu} + \frac{14\nu}{9\mu}$$

where μ and ν are relatively prime positive integers. Determine all pairs (μ, ν) for which A is an integer.

Problem 3: Let $\varDelta AB\varGamma$ be an isosceles triangle with $\varGamma A = \varGamma B$. Let (δ_1) and (δ_2) be the bisectors of angles $\angle \varGamma AB$ and $\angle \varGamma BA$ respectively. From vertex $\varGamma$, we drop perpendicular lines (ε_1) and (ε_2) on (δ_1) and (δ_2) which meet the bisectors $(\delta_1), (\delta_2)$ at Z, H respectively. Let $\varTheta, I$ be the points of intersection of (ε_1) and (ε_2) with the line AB respectively. If K, N are the midpoints of the segments $\varTheta H$ and IZ respectively, prove that:

(α) The quadrilateral $ZHI\varTheta$ is an isosceles trapezium.

(β) $(\varGamma KN) = (ZKNH)$

Note: With (T), we denote the area of figure T.

Problem 4: Let α, β be real numbers such that

$$\alpha^3 + \beta^3 + 9\alpha\beta = 27$$

Determine all possible values of $\varSigma = \alpha + \beta$.