Problema 2. Arătaţi că pentru orice numere reale $a, b, c > 0$ are loc

$$\frac{(a+b)^2}{ab} + \frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} \geq 9 + 2\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right).$$

Soluţie: (*Marius Valetin Drăgoi*)
$\sum \frac{(a+b)^2}{ab} \geq 9 + 2 \cdot \sum \frac{a}{b+c} \Leftrightarrow \sum \frac{a^2+2ab+b^2}{ab} \geq 9 + 2 \cdot \sum \frac{a}{b+c} \Leftrightarrow 6 + \sum \frac{a^2+b^2}{ab} \geq 9 + 2 \cdot \sum \frac{a}{b+c} \Leftrightarrow \sum \frac{a^2+b^2}{ab} \geq 3 + 2 \cdot \sum \frac{a}{b+c} \Leftrightarrow \sum \left(\frac{a}{b} + \frac{b}{a}\right) \geq 3 + 2 \cdot \sum \frac{a}{b+c} \Leftrightarrow \sum \left(\frac{a}{b} + \frac{a}{c}\right) \geq 3 + 2 \cdot \sum \frac{a}{b+c}$.
Din CBS avem: $\frac{a}{b} + \frac{a}{c} \geq \frac{(\sqrt{a}+\sqrt{a})^2}{b+c} = \frac{4a}{b+c} \Rightarrow \sum \left(\frac{a}{b} + \frac{a}{c}\right) \geq 4 \cdot \sum \frac{a}{b+c} \geq 3 + 2 \cdot \sum \frac{a}{b+c} \Leftrightarrow 2 \cdot \sum \frac{a}{b+c} \geq 3 \Leftrightarrow \sum \frac{a}{b+c} \geq \frac{3}{2}$. (*Nesbitt*) Adevărat.