Monday, June 26, 2017

Problem 1. Determine all the sets of six consecutive positive integers such that the product of some two of them, added to the product of some other two of them is equal to the product of the remaining two numbers.

Problem 2. Let x, y, z be positive integers such that $x \neq y \neq z \neq x$. Prove that

$$(x + y + z)(xy + yz + zx - 2) \geq 9xyz.$$

When does the equality hold?

Problem 3. Let ABC be an acute triangle such that $AB \neq AC$, with circumcircle Γ and circumcenter O. Let M be the midpoint of BC and D be a point on Γ such that $AD \perp BC$. Let T be a point such that $BDCT$ is a parallelogram and Q a point on the same side of BC as A such that

$$\angle BQM = \angle BCA \quad \text{and} \quad \angle CQM = \angle CBA.$$

Let the line AO intersect Γ at E, $(E \neq A)$ and let the circumcircle of $\triangle ETQ$ intersect Γ at point $X \neq E$. Prove that the points A, M, and X are collinear.

Problem 4. Consider a regular $2n$-gon P in the plane, where n is a positive integer. We say that a point S on one of the sides of P can be seen from a point E that is external to P, if the line segment SE contains no other points that lie on the sides of P except S. We color the sides of P in 3 colors (ignore the vertices of P, we consider them colorless), such that every side is colored in exactly one color, and each color is used at least once. Moreover, from every point in the plane external to P, points of at most 2 different colors on P can be seen. Find the number of distinct such colorings of P (two colorings are considered distinct if at least one side is colored differently).

Language: English

Time: 4 hours and 30 minutes
Each problem is worth 10 points