



CYPRUS MATHEMATICAL SOCIETY
C' SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 9/04/2016

Time duration: 10:00-14:30

Instructions:

1. Solve all the problems showing your work.
 2. Write with blue or black ink. (You may use pencil for figures)
 3. Do not use corrector liquid (Tipp-ex).
 4. Do not use calculators.
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Problem 1: Find all positive integers v such that the numbers $v + 181$ and $v - 288$ are perfect cubes.

(An integer κ is called perfect cube if and only if there exists an integer α such that $\kappa = \alpha^3$)

Problem 2: Given a right triangle $\triangle AB\Gamma$ ($\angle A = 90^\circ$) and its altitude $A\Delta$ (the point Δ lies on $B\Gamma$).

a) Prove that: $AB \cdot A\Gamma = B\Gamma \cdot A\Delta$

b) If the sides α, β, γ of the triangle $\triangle AB\Gamma$ have the property $\alpha = 2\sqrt{\beta\gamma}$, find the angles of the triangle.

Problem 3: If the lengths α, β, γ of the sides of a triangle $\triangle AB\Gamma$ satisfy the inequality

$$\alpha\sqrt{8} + \beta\sqrt{6} + \gamma\sqrt{2} \geq 4\sqrt{\alpha^2 + \beta^2 + \gamma^2},$$

prove that the triangle is a right one.

Problem 4: Three friends crossing a forest found a large box, full of nuts. Because it was late in the night, they decided to sleep and to divide the nuts next morning. One of the three was awake, threw 3 nuts to squirrels, hid the $1/4$ of the remaining nuts and went to sleep. Then one of the two others woke up, threw 3 nuts to squirrels, hid the $1/4$ of the remaining nuts and went to sleep. Next morning the third one woke too early and first of all, threw 3 nuts to squirrels, hid the $1/4$ of the remaining nuts and divided what remained into three bags. When the other two woke up, he told them that these three bags remained, because of the squirrels. Find how many nuts at most they had found last night, if the three bags have less than 40 nuts each.