CYPRUS MATHEMATICAL SOCIETY
B΄ SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 27/02/2016 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work.
2. Write with blue or black ink. (You may use pencil for figures)
3. Do not use corrector liquid (Tipp-ex).
4. Do not use calculators.

Problem 1: Let ν be a positive integer such that $3\nu - 2$ divides $3\nu^2 - 2\nu - 9$ and $2\nu - 1$ divides $2\nu^2 - \nu - 17$. Find all the possible values of the expression:

$$A = \frac{(3\nu^2 - 2\nu - 9)(2\nu^2 - \nu - 17)}{(3\nu - 2)(2\nu - 1)}$$

Problem 2: Let x, y, z be real numbers, satisfying the following relations:

$$\begin{cases} x + y + z = 3 \\ x^2 + y^2 + z^2 = 3 \\ x^3 + y^3 + z^3 = 3 \end{cases}$$

Find the product xyz.

Problem 3: Given triangle $\triangle AB\Gamma$. The internal bisectors of the angles $\angle B, \angle \Gamma$ meet the sides of the triangle $A\Gamma, AB$ at the points Δ, E, respectively. Let K and N be points on the bisectors ΓE and $B\Delta$, respectively, such that $AK \perp \Gamma E$ and $AN \perp B\Delta$. If KN meets AB at the point Z, prove that the triangle $\triangle ZBN$ is isosceles.

Problem 4: Determine all positive integers ν, $\nu \leq 2016$, which can be written as a sum of at least 60 consecutive positive integers.