ΚΥΠΡΙΑΚΗ ΜΑΘΗΜΑΤΙΚΗ ΕΤΑΙΡΕΙΑ

Α΄ ΔΙΑΓΩΝΙΣΜΟΣ ΕΠΙΛΟΓΗΣ ΚΑΤΩ ΤΩΝ 15 1/2 ΕΤΩΝ

«Ευκλείδης»

Ημερομηνία: 23/01/2016 **Ώρα εξέτασης: 10:00-14:30**

ΟΔΗΓΙΕΣ:

1. Να λύσετε **όλα** τα θέματα **αιτιολογώντας** πλήρως τις απαντήσεις σας.
2. Να γράφετε με μπλε ή μαύρο μελάνι. (Τα σχήματα επιτρέπεται με μολύβι)
3. Δεν επιτρέπεται η χρήση διορθωτικού υγρού .
4. Δεν επιτρέπεται η χρήση υπολογιστικής μηχανής.

INSTRUCTIONS:

1. Solve all the problems fully justifying your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid is not permitted.
4. Calculators are not permitted.

Problem 1: Find all triples (α, β, γ) of positive integers, which satisfy

$$\begin{cases} \alpha^3 - \beta^3 - \gamma^3 = 3\alpha\beta\gamma \\ \alpha^2 = 2(\alpha + \beta + \gamma) \end{cases}$$

Problem 2: Given an acute triangle ⊿ABC with ∠B = 2 (∠C) (the angle ∠B is twice the ∠C). From the point A we draw a perpendicular to BC, which intersects BC at the point D. On the extension of AB in the direction of B we get a point E such that BE = BD. We draw the line ED, which intersects the line AC at the point M and the perpendicular from the points A and C to the line ED, which intersect the ED at the points K and L respectively. Prove that
(a) The quadrilateral ALCK is a parallelogram
(b) ∠BAD = ∠KAM

Problem 3: Let N a positive integer number. Prove that the number N^3 can be written as a difference of squares of two positive integer numbers.

Problem 4: For a four-digit number the following apply:

- Each digit is one of the digits 1, 2, 3 and 4.
- Every two consecutive digits are mutually different.
- The thousands digit and the units digit are mutually different.
- The thousands digit is not larger than any other digit.

Find how many such four-digit numbers exist.