CYPRUS MATHEMATICAL SOCIETY
A΄ SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 09/02/2013 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work .
2. Write with blue or black ink . (You may use pencil for figures)
3. Do not use corrector liquid (Tipp-ex).
4. Do not use calculators .

Problem 1 **:** If α, β, γ are rational numbers such that $\alpha \neq \beta \neq \gamma \neq \alpha$, prove that $A = \frac{1}{(\alpha-\beta)^2} + \frac{1}{(\beta-\gamma)^2} + \frac{1}{(\gamma-\alpha)^2}$ is a perfect square of rational number.

Problem 2 **:** 2013 real numbers are written in a row. If α, β, γ are three consecutive ones, then $\beta = \frac{2\alpha\gamma}{\alpha+\gamma}$. The first number is $\frac{1}{10}$ and the last one is $\frac{1}{603}$.

Find the 1001^{st} number.

Problem 3 **:** Given a triangle with $\angle A = 60^0$. We draw its bisectors $B\Delta$ and ΓE. Prove that $BE + \Gamma\Delta = B\Gamma$.

Problem 4 **:** Let α, β, ν are positive integers and suppose that $B = \nu^\beta + 1$, $A = \nu^\alpha + 1$. Prove that the number A is divisible by the number B if and only if there exists odd number k, such that $\alpha = k\beta$.