CYPRUS MATHEMATICAL SOCIETY

B΄ SELECTION COMPETITION

FOR UNDER 15 1/2 YEARS OLD

«Euclidis»

Date: 13/03/2010 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work .
2. Write with blue or black ink . (You may use pencil for figures)
3. Do not use corrector liquid (Tipp-ex).
4. Do not use calculators .

Problem 1 : Find all integers $\nu \neq 3$ for which the number $\nu - 3$ divides the number $\nu^3 - 3$.

Problem 2 : In a triangle $AB\Gamma$ the angle $\angle A = 45^o$, M is the midpoint of $B\Gamma$ and $B\Delta, \Gamma E$ the heights of the triangle. Prove that the triangle ΔME is a right triangle and isosceles.

Problem 3 : Find the real numbers x, y, z for which satisfy the following equation

$$x^2 + y^2 + z^2 + xy + yz + zx + \frac{3}{2} = 2(x + y + z)$$

Problem 4 : A triangle $AB\Gamma$ is given.

a) In the inner part of the side $A\Gamma$ we take ν different points $B_1, B_2, \dots, B_\nu$ and in inner part of the side AB we take also ν Different points $\Gamma_1, \Gamma_2, \dots, \Gamma_\nu$. Show that the 2ν line segments $BB_i \; and \; \Gamma\Gamma_i, i = 1,2,3, \dots, \nu$, divide the inner part of the triangle in $(\nu + 1)^2$ regions.

b) In the inner part of the side $B\Gamma$ we chose the point Ω such that the $A\Omega$ and any two of the segments $BB_i \; and \; \Gamma\Gamma_i, i = 1,2,3, \dots, \nu$, are not passing through the same point. Find the number of regions that are formed in the inner part of the triangle from all the segments above $BB_i and \; \Gamma\Gamma_i$ and $A\Omega$.