**CYPRUS MATHEMATICAL SOCIETY**

**B΄ SELECTION COMPETITION**

**FOR UNDER 15 1/2 YEARS OLD**

# «Euclidis»

**Date:** 13/02/2010 **Time duration: 10:00-14:30**

**Instructions:**

1. Solve all the problems showing your work .
2. Write with blue or black ink . (You may use pencil for figures)
3. Do not use corrector liquid (Tipp-ex).
4. Do not use calculators .

***Problem 1 :*** Find the smallest positive integer , which can be written as a sum of $9, 10\ and\ 11$ consecutive positive integers.

***Problem 2 :*** The following numbers are given:
$A = \underbrace{888\ldots8}_{2009-digits}$ , $B = \underbrace{444\ldots4}_{2009-digits}$ , $\Gamma = \underbrace{333\ldots3}_{2008-digits}$ , and $\Delta = \underbrace{666\ldots67}_{2008-digits}$. Compare the products $X = A \cdot \Gamma\ and\ \Psi = B \cdot \Delta$ and find the difference $|X - \Psi|$.

***Problem 3 :*** Find all the integer solutions of the equation:

$$\frac{4x + y + 4\sqrt{xy} + 48}{2\sqrt{x} + \sqrt{y}} = 14$$

***Problem 4 :*** A circle with centre $O$ and radius $R$ is given. Let $AB$ a diameter of the circle and $\Gamma$ an arbitrary point of the circle. From the point $\Gamma$ we draw a perpendicular to $AB$ and we let $\Delta$ the point of its intersection with the diameter. With centre at $\Gamma$ and radius $\Gamma\Delta$, we draw a circle that meets the previous circle at the points $E\ and\ Z$. Prove that:
α) The line segment $EZ$ intersects the line segment $\Gamma\Delta$ at its midpoint.
β) The triangles $E\Gamma Z\ and\ E\Delta Z$ have equal areas.