CYPRUS MATHEMATICAL SOCIETY
B΄ SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 4/03/2017 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work.
2. Write with blue or black ink. (You may use pencil for figures)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.

Problem 1: Given $\alpha, \beta, \gamma \in \mathbb{R}$ such that

$$\alpha + \beta + \gamma = 2017 \quad \kappa\alpha\iota \quad \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 3$$

Prove that at least one of the numbers

$$\frac{\alpha^2 + \beta^2}{\alpha\beta}, \qquad \frac{\beta^2 + \gamma^2}{\beta\gamma}, \qquad \frac{\gamma^2 + \alpha^2}{\alpha\gamma}$$

is greater than or equal to 2016.

Problem 2: Find all the pairs of integers (α, β) satisfying the equation

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{2017}$$

Problem 3: Consider an obtuse triangle $\varDelta AB\varGamma$ with $\angle A\varGamma B > 90^0$ inscribed in a circle (O, R). Draw the altitude $\varGamma K$ of the triangle and let $\varDelta$ be the second point where the line $\varGamma K$ meets the circle (O, R). Through the point $\varDelta$ draw the perpendicular to the line $\varGamma B$, which intersect the line AB at Z. Prove that:

α) The perpendicular from B to $\varGamma Z$ passes through the point $\varDelta$.

β) $\varGamma A = \varGamma Z$

γ) $KA^2 + KB^2 + K\varGamma^2 + K\varDelta^2 = 4R^2$

Problem 4: Andreas, Vasilis, George, Dimitris and Efthimios are exchanging passes with a ball under the following rules:

- Vasilis and George **never** exchange passes with each other.
- Dimitris never passes the ball to Efthimios but Efthimios might pass the ball to Dimitris.
- Efthimios never passes the ball to Andreas but Andreas might pass the ball to Efthimios.

Find the number of ways the above boys can exchange five passes, given that the ball starts from Andreas and comes back to Andreas after the 5th pass. For example one way is the following:

$$Andreas \longmapsto George \longmapsto Andreas \longmapsto Vasilis \longmapsto Dimitris \longmapsto Andreas$$

Note: It has to be taken for granted that nobody passes the ball to himself.