

1. The radii of three concentric circles are 1, 2, and 3 units. A point is marked on each circle such that they form a regular triangle. What may be the length of the side of the triangle?
2. A line of one hundred boys and another line of one hundred girls are standing facing one another. Every boy chooses a girl (it is allowed for more than one boy to choose the same girl) and walks up to her along the shortest path. In doing so, their paths do not cross. Then the boys go back to their places, and now the girls do the same, making sure that their paths do not cross as they are walking up to the chosen boy. Prove that there is a girl and a boy who chose each other.
3. There are 98 sticks lying on the table, their lengths are 1, 2, 3, ..., 98 units. Ann and Bill play the following game: With Ann starting the game, they take turns removing one stick of their choice. The game ends when there remain exactly three sticks on the table. Ann wins if the three sticks can form a triangle. Otherwise, Bill wins. Which player has a winning strategy?
4. There are 99 sticks lying on a table, their lengths are 1, 2, 3, ..., 99 units. Andrea and Bill play the following game: they take turns removing one stick of their choice. Andrea starts the game. The game ends when there are exactly three sticks remaining on the table. If it is possible to make a triangle out of the three sticks then Andrea wins. Otherwise, Bill is the winner. Who has a winning strategy?
5. Prove that in any set of seven different positive integers there are three numbers such that the greatest common divisor of any two of them leaves the same remainder when divided by three.
6. The non-negative real numbers a, b, c, d add up to 1. Prove the inequality $|ab - cd| \leq \frac{1}{4}$.
7. Is there any order of the numbers 1, 1, 2, 2, ..., 1998, 1998 in which there are exactly n numbers placed between the two copies of n , for every $1 \leq n \leq 1998$?
8. There are given $2n + 1$ odd positive numbers, none of which is greater than $6n$. Prove that one of these numbers divides another one.
9. A cuboid is built out of $5 \times 10 \times 20$ -cm bricks without leaving any gaps between them. Prove that the same cuboid can also be built out of the same bricks with all edges of equal lengths being parallel.
10. Prove that if a, b, c, d are integers and $a + b + c + d = 0$ then $2(a^4 + b^4 + c^4 + d^4) + 8abcd$ is a square number.
11. Prove that if the natural numbers a and b only differ in the order of their digits then the sum of the digits in the numbers $5a$ and $5b$ is the same.
12. Fie ABC un triunghi neisoscel. Fie ω cercul înscris și I centrul acestuia. Notăm cu M, N și P punctele de contact ale cercului cu laturile BC, CA , respectiv AB . Fie J punctul de intersecție a dreptelor MN și IC . Dreapta PJ intersectează ω în K . Demonstrați că semidreapta $(CI$ este bisectoarea unghiului $\sphericalangle PCK$.
13. În interiorul pătratului $ABCD$ se consideră două puncte, M și N , astfel încât $m(\sphericalangle MAN) = m(\sphericalangle MCN) = 45^\circ$, $M \in Int(\sphericalangle NAD)$. Suprafețele triunghiurilor MAD, MCN și NAB se colorează cu roșu, iar suprafețele triunghiurilor MCD, MAN și NCB se colorează cu albastru. Arătați că suprafața colorată cu roșu și suprafața colorată cu albastru au aceeași arie.
14. Fie $k \in \mathbb{R}$, fixat. Să se determine mulțimea valorilor expresiei $\frac{(a + b + c)^3}{abc}$, unde a, b, c sunt numere reale nenule astfel încât $\frac{b + kc}{a} = \frac{c + ka}{b} = \frac{a + kb}{c}$.