TALMEŞ BALMEŞ

1. Consider a company of $n \geq 4$ people, where everyone knows at least one other person, but everyone knows at most $n-2$ of the others. Prove that we can sit four of these people at a round table such that all four of them know exactly one of their two neighbors. (Knowledge is mutual.)

2. We are given an acute triangle ABC, and inside it a point P, which is not on any of the heights AA_1, BB_1, CC_1. The rays AP, BP, CP intersect the circumcircle of ABC at points A_2, B_2, C_2. Prove that the circles AA_1A_2, BB_1B_2 and CC_1C_2 are concurrent.

3. If $a, b, c \geq 0$, prove that $\dfrac{a}{1+a} + \dfrac{b}{1+b} + \dfrac{c}{1+c} \geq \dfrac{a+b+c}{1+a+b+c}$.

4. Un dreptunghi 2010×1000 este împărţit în pătrăţele unitate. Construim una din diagonalele dreptunghiului. Câte pătrăţele unitate traversează diagonala? (Pătrăţelele care au un singur punct comun cu diagonala nu vor fi numărate.)

5. Se ştie că 2^{333} este un număr de 101 cifre a cărui primă cifră este 1. Câte dintre numerele 2^k, $1 \leq k \leq 332$, au prima cifră 4?

6. We have placed $n > 3$ cards around a circle, facing downwards. In one step we may perform the following operation with three consecutive cards. Calling the one on the center B, the two on the ends A and C, we put card C in the place of A, then move A and B to the places originally occupied by B and C, respectively. Meanwhile, we flip the cards A and B.

Using a number of these steps, is it possible to move each card to its original place, but facing upwards?