

1. The benches of the Great Hall of the Parliament of Neverenough are arranged in a rectangle of 10 rows of 10 seats each. All the 100 MPs have different salaries. Each of them asks all his neighbours (sitting next to, in front of, or behind him, as well as those four seated diagonally in front of or behind him, i.e. 8 members at most) how much they earn. They feel a lot of envy towards each other: an MP is content with his salary only if he has at most one neighbour who earns more than himself. What is the maximum possible number of MPs who are satisfied with their salaries?
2. Într-un paralelogram, diagonala mai scurtă împarte paralelogramul în două triunghiuri. Se consideră cercul înscris într-unul din aceste triunghiuri și cercul exînscribit tangent diagonalei în celălalt triunghi. Arătați că cele patru puncte de tangență ale celor două cercuri care nu se află pe diagonala paralelogramului sunt coliniare.
3. A positive integer  $n > 2$  is said to be *undivided* if  $1 < k < n$  and  $(k, n) = 1$  imply that  $k$  is a prime. How many undivided numbers greater than 2 are there?
4. A  $23 \times 23$  square is divided into smaller squares of dimensions  $1 \times 1$ ,  $2 \times 2$  and  $3 \times 3$ . What is the minimum possible number of  $1 \times 1$  squares?
5. The forum of this journal on the internet has exactly  $6n$  registered members. Each member sends an e-mail to one other member and receives exactly one mail from one other member. Prove that we can select a group of at least  $2n$  and at most  $3n$  members such that no one in the group sent an e-mail to anyone else.
6. Given that  $a + b \leq c + 1$ ,  $b + c \leq a + 1$ ,  $c + a \leq b + 1$  for the numbers  $a, b, c \geq 0$ , show that  $a^2 + b^2 + c^2 \leq 2abc + 1$ .
7. In the parallelogram  $ABCD$ ,  $2BD^2 = BA^2 + BC^2$ . Show that the circumscribed circle of triangle  $BCD$  goes through one of the points that trisect the diagonal  $AC$ .
8. This problem is a classic: A town is surrounded by a circular wall. There are 12 guards serving on the wall. At twelve noon, each guard leaves his watchpost and starts walking the wall in some direction, at a speed at which it would take exactly one hour to walk around the whole town. If two guards meet, they both turn around immediately and walk at the same speed in the opposite direction. Prove that at twelve midnight each guard will be back at his watchpost.
9. A, B, C, D, E and F are a group of six people.  $n$  bars of chocolate given to the group in the following way: Everyone gets at least one, A gets less than B, B gets less than C, C gets less than D, D gets less than E, and finally, F gets the most. The members of the group know these conditions, they know the value of  $n$ , and of

course, they know how many bars of chocolate they were given themselves. They have no other information available for them. What is the smallest possible value of  $n$  for which it is possible to give them the bars of chocolate so that no one can tell how many bars of chocolate everyone has?

**10.** Let  $a \geq b \geq c > 0$ . Prove that  $\frac{a^2 - b^2}{c} + \frac{c^2 - b^2}{a} + \frac{a^2 - c^2}{b} \geq 3a - 4b + c$ .