

1. Fie  $a, b, c$  numere reale pozitive cu  $a^2 + b^2 + c^2 = 1$ . Aflați minimul expresiei

$$\frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b}.$$

2. Fie  $ABCD$  un paralelogram. Arătați că cercurile Euler ale triunghiurilor  $ABC$  și  $ADC$  sunt tangente.

3.  $I$  e punctul lui Torricelli în triunghiul  $ABC$  (adică punctul situat în interiorul triunghiului pentru care  $m(\sphericalangle AIB) = m(\sphericalangle BIC) = m(\sphericalangle CIA) = 120^\circ$ ). Demonstrați că drepte Euler ale triunghiurilor  $ABI, BCI$  și  $CAI$  sunt concurente.

4. An interior point of a regular  $2n$ -gon is connected to each vertex to form  $2n$  triangles. The triangles are colored red and blue alternately. Prove that the total blue area equals the total red area.

5. Prove that every natural number not divisible by 10 can be multiplied by an appropriate natural number, such that the product is a palindromic number in decimal notation.

6. Each member of the sequence  $a_1, a_2, \dots, a_{2n+1}$  is either 2, 5 or 9. No two consecutive members are equal, and  $a_1 = a_{2n+1}$ . Prove that  $a_1a_2 - a_2a_3 + a_3a_4 - a_4a_5 + \dots - a_{2n}a_{2n+1} = 0$ .

7. All the numbers have fallen down one by one from the face of an old wall-clock. Prove that if we replace the numbers, in any order, on the empty face of the clock, there will be three consecutive numbers among them that add up to 20 at least. Is it always true that there will also be a sum greater than 20?

8. Is it true that every integer has at least as many positive divisors of the form  $4k + 1$  as that of the form  $4k - 1$ ?

9. Two players alternately mark the fields of a  $5 \times 5$  board. The one who moves first always writes one X sign, while the second player always writes two O signs. The one who first completes a row or a column of the board with her signs, wins the game. How can the second player win the game?

10. Ann wrote 32 integers on a large sheet of paper, and covered each number with a card. Then he told Bob that if he chose 7 cards, she would tell him whether the sum of the 7 covered numbers is odd or even. At least how many times did Bob have to choose 7 cards in order to find out if the sum of all 32 numbers on the sheet was odd or even?