Junior Balkan MO 2007

Shumen, Bulgaria

- 1. Let a be positive real number such that $a^3 = 6(a+1)$. Prove that the equation $x^2 + ax + a^2 6 = 0$ has no real solution.
- 2. Let ABCD be a convex quadrilateral with $\angle DAC = \angle BDC = 36^{\circ}$, $\angle CBD = 18^{\circ}$ and $\angle BAC = 72^{\circ}$. The diagonals and intersect at point P. Determine the measure of $\angle APD$.
- 3. Given are 50 points in the plane, no three of them belonging to a same line. Each of these points is colored using one of four given colors. Prove that there is a color and at least 130 scalene triangles with vertices of that color.
- 4. Prove that if p is a prime number, then $7p + 3^p 4$ is not a perfect square.