Problem 1

Find the least possible sum of the digits of the decimal representation of $3n^2+n+1$, where $n \geq 2$ is an integer.

Problem 2

Let $n \geq 3$ be an integer and $M = \{1, 2, \ldots, 2n-1\}$. In each unit square of a $n \times n$ table we write an element of M.

We call such a filling *good* if for every i, $1 \leq i \leq n$, on the line i and the column i of the table one can find all the elements of M.

a) Find an integer n for which there exists a *good filling* of a $n \times n$ table.

b) Prove that there does not exist a *good filling* of a 2017×2017 table.

Problem 3

Given an acute triangle ABC with orthocentre H, let A_1, B_1, C_1 be the feet of the altitudes drawn from A, B, respectively C. Denote by P the intersection of the line AB with the perpendicular from H onto the line A_1C_1, and by Q the intersection of the line AC with the perpendicular from H onto the line A_1B_1.

Prove that the midpoint of the segment $[PQ]$ belongs to the perpendicular from A onto B_1C_1.

Problem 4

Find all triples of positive integers (x, y, z) such that x and y are coprime and $x^4 + y^4 = 2z^2$.